CHAPTER 14: GRAVITATION

Gravitation is the mutual attraction of all masses in the universe. The concepts associated with planetary motions were developed by Kepler. Newton explained why Kepler's Laws worked in terms of gravitation.

MAJOR CONCEPTS/ EQUATIONS

- Newton's Law of Gravitation
- Superposition
- Shell Theorems
- Kepler's Law
- Gravitational Potential Energy

- $F = G \dfrac{m_1 m_2}{r_2}$ (Newtons' law of gravitation)
- $F_1 = \sum_{i=2}^{n} F_{1i}$ (14-4)
- $F = ma_g$ (14-11)
- $U = \dfrac{GMm}{r}$ (Gravitational potential energy)
- $U = -\left(\dfrac{Gm_1 m_2}{r_{12}} + \dfrac{Gm_1 m_2}{r_{13}} + \dfrac{Gm_2 m_2}{r_{23}}\right)$ (14-21)
- $T^2 = \left(\dfrac{4\pi^2}{GM}\right) r^3$ (law of periods)
- $U = -\dfrac{GMm}{r}$ and $K = \dfrac{GMm}{2r}$ (14-20, 14-40)

IMPORTANT NOTES

- Earth is not uniform, and is not a sphere
- Earth is rotating
- **Kepler's Laws:** 1. All planets move in elliptical orbits, with the Sun at one focus. 2. A line that connects a planet to the Sun sweeps out equal areas in equal times. 3. The square of the period of any planet is proportional to the cube of the semimajor axis of its orbit.
- Any particle in the universe attracts any other particle with a **gravitational force**
- A uniform shell of matter exerts no gravitational force on a particle located inside it.

PROBLEM TYPES

- Motion gravitational acceleration
- Free- fall acceleration and weight
- Energy in Planetary Motion
- Gravitational potential energy
- Potential energy of a system
- Kepler's Laws

1. A mass M is split into two parts, m and M-m, which are then separated by a certain distance. What ratio m/M maximizes The gravitational force between the parts?

We need to maximize the function

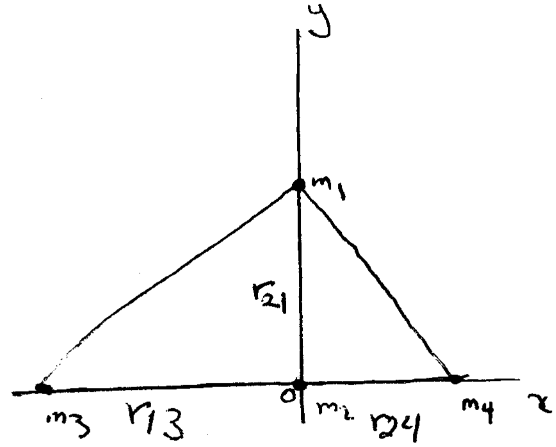
$f(m) = m(M-m)$.

This is done by setting:

$df(m)/dm = M - 2m = 0$

$\therefore \frac{m}{M} = \frac{1}{2}$.

2. Four spheres, with masses m1 = 400 kg, m2 =350 kg, m3 = 2000, and m4 = 500 kg, have (x, y) coordinates of (0, 50 cm), (0,0), and (40 cm,), respectively. What is the net gravitational force F2 on m2 to the other masses?



The net Fore is givenby

$$F_2 = G m_2 \left(\frac{m1}{r_{21}^2} j - \frac{m3}{r_{23}^2} i + \frac{m4}{r_{24}^2} i \right)$$

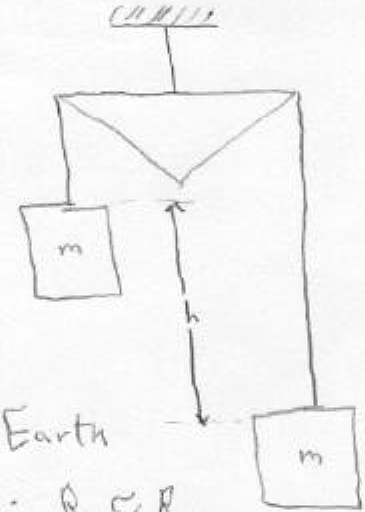
$$= \left(6.67\times10^{-11}\ m^2/kg\cdot s^2\right) (350) \left[\frac{400}{(.5m)^2} j - \frac{2000}{(.8)^2} i + \frac{500}{(0.4m)^2} i \right]$$

$$= 3.7\times10^{-5}\ N j$$

3. Calculate the gravitational acceleration on the surface of the Moon from Values for the mass and radius of the Moon?

$$g_m = G \frac{M_m}{R^2_m} = \left(6.67 \times 10^{-11}\ m^3/kg \cdot s^2\right) \frac{(7.36 \times 10^{22} kg)}{(1.74 \times 10^6 m)^2} = 1.62\ m/s^2$$

4. In Fig. Identical masses m hang from strings on a balance at the surface of Earth. The strings have negligible mass and differ in length by h. Assume that Earth is spherical. With density 5.5 g/cm3. (a) Show that the difference ΔW in the weight, due to the one mass being closer to Earth than the other, is 8ΠGρmh/3. (b) Find the difference in length that will give a ratio ΔW/W = 1*10-6, where W is either weight.



a) $$\Delta W = G\frac{mM_e}{R^2} - G\frac{mM_e}{(R+h)^2}$$

$$= \frac{GmM_e}{R^2}\left[1-\left(1+\frac{h}{R}\right)^2\right]$$

$$\simeq \frac{GmM_e}{R^2}\left[1-1+\frac{2h}{R}\right]$$

$$= 2h\,\frac{GmM_e}{R^3}$$

Where R is distance from the center of Earth to the center of mass of the lower mass: $R \cong R_e$

Thus $\frac{M_e}{R^3} \simeq \frac{M_e}{R_e^3} = \left(\frac{4\pi}{3}\right)\rho$, and

$$\Delta W = 2h\,Gm\left(\frac{4\pi\rho}{3}\right) = \frac{8\pi G\rho m h}{3}$$

b) Let $\frac{\Delta W}{W} = 1.0\times10^{-6}$ then:

$$\frac{\Delta W}{W} = \frac{8\pi\rho m h/3}{mg}$$

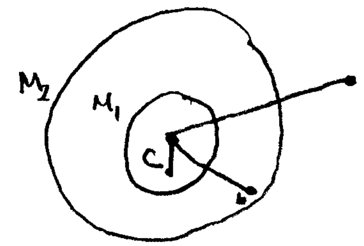
which gives

$$h = \left(\frac{3g}{8\pi G\rho}\right)\left(\frac{\Delta W}{W}\right)$$

$$\simeq \frac{3(9.80)(1.0\times10^{-6})}{8\pi(6.67\times10^{-11})(5.5\times10^{3})} = 3.2\text{ m}$$

5. Two concentric shells of uniform density having masses M1, and M2 are situated as shown in Fig 14-36. Find the force on a particle of mass m when the particle is located at (a) r + a, (b) r = b and (c) r= c. The distance r is measured from the center of the shells.

Use Newton's shell theorem.



(a) $F_a = G(M_1 + M_2)\ m/a^2.$

(b) $F_b = \dfrac{G M_1 m}{b^2}$

(c) $F_c = 0.$

10^3 10^4

6. The mean diameters of Mars and Earth are 6.9 * E3 km and 1.3 *E4 km, respectively. The mass of Mars in 0.11 times Earth's mass. (a) What is the ratio of the mean density of Mars to that of Earth? (b) What is the value of g on Mars? (c) What is the escape speed on Mars?

(a) $$\frac{\rho_m}{\rho_e} = \frac{M_m/V_m}{M_e/V_e} = (0.11)\left(\frac{1.3\times10^4}{6.9\times10^3}\right) = 0.74.$$

(b) Use Eq 14-12:

$$a_g = \frac{GM_m}{R^2_m} = \frac{M_m}{4\pi R^3_m/3}\cdot\frac{4\pi G R_m}{3}$$

$$= \frac{4\pi}{3} G\rho_e R_e \left(\frac{\rho_m}{\rho_e}\right)\left(\frac{R_m}{R_e}\right) = g\left(\frac{\rho_m}{\rho_e}\right)\left(\frac{R_m}{R_e}\right)$$

$$= (9.8)(0.736)\left(\frac{6.9\times10^3}{1.3\times10^4}\right) \approx 3.8\ m/s^2$$

(c) Use Eq 14-26

$$V_m = \sqrt{\frac{2GM_m}{R_m}} = \sqrt{\frac{2GM_e}{R_e}}\sqrt{\left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)}$$

$$= V_e\sqrt{\left(\frac{M_m}{M_e}\right)\left(\frac{R_e}{R_m}\right)} = (11.2)\sqrt{(0.11)\left(\frac{1.3\times10^4}{6.9\times10^3}\right)}$$

$$= 5.1\ km/s$$

7. A 20 kg mass is located at the origin, and a 10 kg mass is located on the x-axis at x = 0.80m. The 10 kg mass is released from rest while the 20 kg mass is held in place at the origin. (a) What is the gravitational potential energy of the two-mass system immediately after the 10 kg mass is released? (b) What is the kinetic energy of the 10 kg mass after it has moved .20 m toward the 20 kg mass?

(a)

$$U_i = -\frac{G m_1 m_2}{x} = -\frac{(6.67\times10^{-11})(20)(10)}{0.8} = -1.67\times10^{-8}\,J$$

(b) Since

$$U_f = -\frac{G m_1 m_2}{x'} = U_i \frac{x}{x'}$$

$$= -(1.67)(10^{-8})\left(\frac{0.8}{0.8-0.2}\right) = -2.23\times10^{-8}\,J$$

We have

$$K_f = -\Delta U = U_i - U_f = -(1.67)(10^{-8}\,J) + 2.23(10^{-8})\,J$$

$$= 5.6\times10^{-9}\,J$$

8. A satellite is placed in a circular orbit with a radius equal to one-half the radius of the Moon's orbit. What is its period of revolution in lunar months? (A lunar month is the period of revolution of the Moon.)

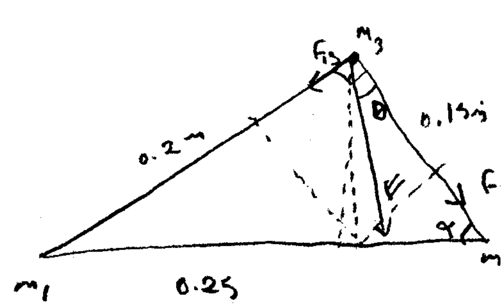
Since $T \propto r^{3/2}$, the period of the satellite is

$$T_s = T_m \left(\frac{r_s}{r_m}\right)^{3/2} = T_m \left(\frac{1}{2}\right)^{3/2} = 0.35\, T_m,$$

Which is about 0.35 lunar months

9. Two spheres with masses m1 = 800kh and m2 = 600kg are separated by .25m. What is the net gravitational force (in both magnitude and direction) from them on a 0.20 kg sphere located 0.20m from m1 and 0.15 m from m2?

Let the forces exterted by m_2 and m_3 on m_1 be F12 and F13 respectively.



$$F = |F_{12} + F_{13}|$$

$$= \sqrt{(F_{12})^2 + (F_{13})^3 + 2F_{12}F_{13}\cos\phi}$$ where the angle θ between F12 and F13 is given by Cosin theorem

$$\cos\theta = \frac{[(0.25)^2 - (0.20)^2 - (0.15)^2]}{[2(0.25)(0.15)]} = 0$$

$$F = \sqrt{(F12)^2 + (F13)^2}$$

$$= (6.67\times10^{-11})(2.0)\sqrt{\frac{800^2}{0.20^4} + \frac{600^2}{0.15^4}} = 4.4\times10^{-6}N$$

$$\theta = \tan^{-1}\left(\frac{F12}{F13}\right) = \tan^{-1}\left(\frac{\frac{800}{.2^2}}{\frac{600}{.15^2}}\right) = 37°$$

Since $\alpha = \tan^{-1}\left(\frac{0.15}{0.20}\right) = 37° = \theta$ F is perpendicular to, and points toward, the line joining m_1 and m_2.

10. A projectile is fired vertically from Earth's surface with an initial speed of 10 km/s. Neglecting air drag, how far above the surface of earth will it go?

Use Conservation of energy for projectile of mass m and intial speed v_i rising to max height of h:

$$\Delta K = -\frac{1}{2} m v_i^2 = -\Delta U = -\left[-\frac{G m M_e}{R_e + h} - \left(-\frac{G m M_e}{R_e} \right) \right]$$

Solve for h:

$$h = \frac{v_i^2 R_e^2}{2 G M_e - v_i^2 R_e}$$

$$= \frac{(10^4)^2 (6.37 \times 10^6)^2}{2(6.67 \times 10^{-11})(5.98 \times 10^{24}) - (10^4)^2 (6.37 \times 10^6)}$$

$$= 2.6 \times 10^7 \, m = 2.6 \times 10^4 \, km$$