Oscillations

Oscillations are motions that repeat themselves. Oscillations are not confined to material objects; it is usually dampened in the real world. The motion dies out gradually transferring mechanical energy to thermal energy by the action of frictional forces.

Simple Harmonic Motion

Frequency

Number of oscillations that are completed each second. f is the symbol for frequency and its SI unit is hertz (Hz).

1 hertz = 1 s-1 (oscillation per second)

Period

Time for one complete oscillation.

$T=1/f$

Simple Harmonic Motion (SHM)

Any motion that repeats itself at regular intervals. The displacement x of the particle from the origin is given the function of time.

$x=Xm\cos(\omega t+\phi)$

Xm is the amplitude of the displacement; the quantity $(\omega t+\phi)$ is the phase of the motion and ϕ is the phase constant. The angular frequency ω is related to the period and frequency of the motion by

$\omega=2\pi/T=2\pi f$ (angular frequency)

-Velocity of SHM

Velocity of a particle moving with simple harmonic motion

$v(t)=-\omega Xm\sin(\omega t+\phi)$

Positive quantity ωXm is the velocity amplitude of the motion.

-Acceleration of SHM

$a(t)= -\omega^2 Xm\cos(\omega t+\phi)$

Positive quantity of $\omega^2 Xm$ is the acceleration amplitude of the motion.

The Force Law for Simple Harmonic Motion

Hooke's Law

$F=-kx$

For a spring, the spring constant being

$K=m\omega^2$

**Simple harmonic motion is the motion executed by a particle of mass m subject to a force that is proportional to the displacement of the particle but opposite in sign.

The block-spring system forms a linear simple harmonic oscillator where F is proportional to x rather than some other power of x. The angular frequency ω of the simple harmonic motion of the block is related to the spring constant k and the mass m of the block, yielding

$\omega=(k/m)^{1/2}$ (angular)

Combining $2\pi f$ and $\omega=(k/m)^{1/2}$ we can find the equation for the period of the linear oscillator.

$T=2\pi$ m/k (period)

Energy In Simple Harmonic Motion

Potential energy of a linear oscillator is associated entirely with the spring. Its value depends on how much the spring is stretched or compressed.

$U(t)=1/2kx^2=1/2kx_m^2\cos^2(\omega t+\phi)$

The kinetic energy of the system is associated entirely with the speed of the block

$K(t)=1/2mv^2=1/2kx_m^2\sin^2(\omega t+\phi)$

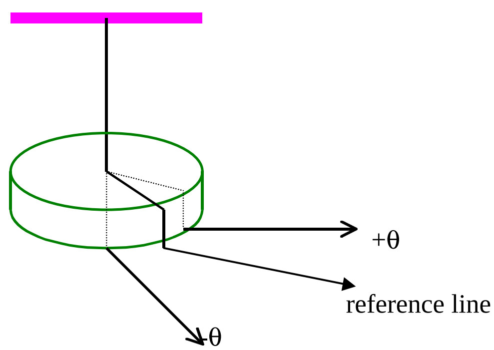
For any angle α

$\cos^2\alpha + \sin^2\alpha=1$

Mechanical Energy

$E=U+K=1/2kx_m^2$

TORSION PENDULUM
(angular simple harmonic oscillator)



torque = -(kappa) x θ

kappa is the torsion constant, which is dependant on length, diameter, and material of the suspension wire. The torque equation is the angular form of Hooke's law.

period = 2 (π) x $\sqrt{\text{inertia / kappa}}$

The disc will rotate back and forth due to the twisting of the suspension wire. Twisting the wire will create potential energy, similar to compressing a spring.

THE SIMPLE PENDULUM

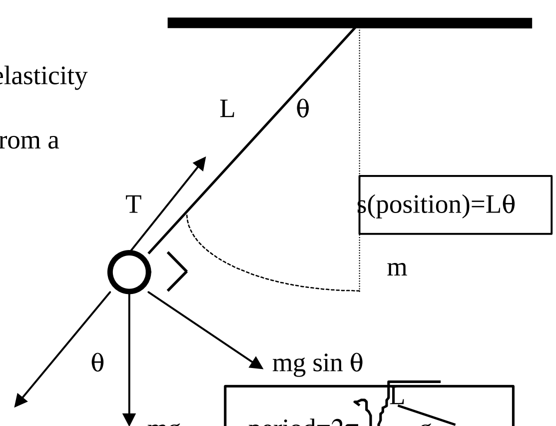
Here, the repetitive motion is due to gravity, not elasticity

A simple pendulum is a bob of mass m swinging from a stretcheable massless string of length L.

The element of inertia is the mass and the element of repetition is due to gravity

The tangential component is a restoring force acting opposite the displacement of the bob

period is independent of mass



period=$2\pi\sqrt{L/g}$

THE PHYSICAL PENDULUM or realistic pendulum.

$\tau=-(mg\sin\theta)(h)$ where h is the distance from the point of suspension to the center of mass torque always acts to reduce the angle to zero

$$\text{period}=2\pi\sqrt{\frac{I}{mgh}}$$

A physical pendulum will not swing if hung by its center of mass because h will equal zero.

free fall acceleration can be found in terms of L and period T

$$g=\frac{8\pi^2 L}{3T^2}$$

Simple Harmonic Motion and Uniform Circular Motion

*The position versus time graph of SHM appears like this

*If one flip this position versus time graph on its side, it acts in the same way as uniform circular motion. SHM is UCM viewed edge-on.

Damped Harmonic Motion

*SHM loses mechanical energy because of drag, friction, thermal, sound, etc.

Damping force (Fd) is the damping constant times the velocity of the oscillator.

The displacement of the damped oscillator is given by

X(t)=Xm^e bt/2m[cos(ω′t+ϕ)]

ω′ (angular frequency)= (k/m b ²/4m²)½

If b<<(km)½ then ω′≈ ω

E=1/2kxm²^e bt/m

Forced Oscillation and Resonance

ùd=ù where ùd⇒ external driving force with angular frequency

⇓

resonance   ù⇒ natural angular frequency of system being acted on

resonance⇒ velocity amplitude (Vm) and amplitude (Xm) of system is greatest

$x = .1m$
$v = -13.6$ m/s
$a = -123$ m/s²
$k = 400$ N/m

$\omega = -a/x$
$= -(-123 \text{ m/s}^2 / .1\text{m})$
$\omega = 35.07$ rad/s

a. $f = \omega/2\pi$
$= 35.07 \text{ rad/s} / 2\pi$
$= 5.58$ Hz

$\tan \phi = v(0)/(x(0)$
$= -13.6 \text{ m/s} / (35.07 \text{ rad/s})(.1\text{m})$
$= -3.88$
$\phi = -75.55°, 104.45°$

b. $Xm = x(0)/\cos \phi$
$= .40$ m

c. $\omega^2 = k/m$
$m = k/\omega^2$
$= 400 \text{ n/m} / (35.07 \text{ rad/s})^2$
$m = .33$ kg

$m = 2.00$kg
$k = 100$ N/m
$t = 1.00$s
$x = .129$m
$v = 3.415$ m/s

a. $\omega = (k/m)^{1/2}$
$= (100\text{N/m} / 2.00 \text{ kg})^{1/2}$
$= 7.07$ rad/s

$V = -\omega Xm$
$3.415 = (-7.07\text{rad/s})Xm$
$Xm = .50$m

Block of mass 20kg is connected to a spring with a constant of 65 N/m. It is pulled a distance of 20m from its equilibrium position of x=0 on a frictionless surface and released from rest at t=0.

a) What is the angular frequency and period?
b) What is the amplitude?
c) What is the speed of the oscillating block?

a. $\omega = (k/m)^{1/2}$
$= (105 \text{ N/m} / 20 \text{ kg})^{1/2}$
$\omega = 2.29$ rad/s

$T = 2\pi/\omega$
$= 2\pi / 2.29$ rad/s
$= 2.74$ s

b. 20 m
*Since the block is released from equilibrium position of 0, it has kinetic energy of 0. Its maximum displacement is 20 m.

c. $V_m = \omega X_m$
$= 2.29$ rad/s(20 m)
$= 45.8$ m/s

52. A FLAT UNIFORM CIRCULAR DISK HAS A MASS OF 3.00KG AND A RADIUS OF 70.0 CM. IT IS SUSPENDED IN A HORIZONTAL PLANE BY A VERTICAL WIRE ATTACHED TO ITS CENTER. IF THE DISK IS ROTATED 2.50rad ABOUT THE WIRE, A TORQUE OF 0.0600 N·m IS REQUIRED TO MAINTAIN THE DISK IN POSITION. CALCULATE a) the rotational inertia of the disk about the wire, b) the torsion constant c) the angular frequency of this torsion pendulum when it is set oscillating.

a) $I=(1/2)mr^2$-------> (1/2) ·3.00kg ·0.7m^2= 0.735 kg m²

b) $\tau=-\kappa\theta$, so $\kappa=-\tau/\theta$-------> -(0.0600Nm/-2.50rad)= 0.024 N m/rad

c) $\omega=2\pi/T$, $T=2\pi\sqrt{(I/\kappa)}$ so, $\omega=1/\sqrt{I/\kappa}$ ————>ω=0.181rad/s

55. THE BALANCE WHEEL OF A WATCH OSCILLATES WITH AN ANGULAR AMPLITUDE OF Pi rad AND A PERIOD OF 0.500 s. FIND a) the max angular speed of the wheel b) the angular speed of the wheel when its displacement is pi/2 rad c) the angular acceleration of the wheel when its displacement is pi/4 rad.

a) $v(t)=-\omega x \sin(\omega t + \theta)$, $\theta=0$, $\omega=2\pi/T=4\pi$, t= π rad, so $v(t)=-4\pi^2 \sin(4\pi t)$ the maximum velocity is 39.5 rad/s (use graph)

b) use $x(t)=\pi/2=x \cos(\omega t+\theta) =\pi \cos(4\pi t)$, so t = 4.65s and $v(t)=-4\pi^2 \sin(4\pi t)$=34.2 rad/s

c) use $x(t)=\pi/4=x \cos(\omega t+\theta) =\pi \cos(4\pi t)$, so t = 6.01s and $a(t)=-\omega^2 x \cos(\omega t)=$
$-16\pi^3 \cos(4\pi t)$=124rad/s

A ROCK CLIMBER FALLS FROM AN OVERHANG AND IS CAUGHT BY HIS ROPE 4.0 METERS FROM WHERE IT IS ATTACHED TO THE CLIFF. THE CLIMBER WEIGHS 160 LBS. ASSUMING THE ROPE IS WEIGHTLESS AND UNSTREACHABLE, a) find the period of oscillation for the system. IF A GUST OF WIND BLOWS THE CLIMBER TO AN ANGLE OF 30 DEGREES TO THE VERTICLE b) find the restoring torque of the system

a)$T=2\pi\sqrt{L/g} =2\pi\sqrt{4.0/9.8}$= 4.01s

b)160 lbs·0.4536kg/lbs =72.58kg, $\tau=-(mg \sin\theta)(h)$=-(72.58·9.8 sin 30)(4.0)=1422.5Nm

A MAN HANGS HIS BICYCLE FROM A HOOK ON THE CEILING IN HIS GARAGE, CAUSING IT TO SWING BACK AND FORTH. IF THE THE BIKE IS 1.1 METERS LONG, AND THE BIKE IS 1.45 KILOGRAMS a)find its rotational inertia b) IF THE DISTANCE FROM THE POINT OF SUSPENSION TO THE CENTER OF MASS IS 0.9METERS, find the period. c) how would the period change if h was 0.35 .

a) use the rotational inertia for a thin rod. $I=(1/3)\,mL^2=1.45(1.1^2)(1/3)=0.58$

b)$T=2\pi\sqrt{I/(mgh)}=2\pi\sqrt{(0.58)/(1.45\cdot 9.8\cdot 0.9)}=1.34$ s

c) $2\pi\sqrt{(0.58)/(1.45\cdot 9.8\cdot 0.35)}=2.15$s, so the period of oscillation is longer when you decrease h, gravity, or weight.

For this damped oscillator, m=300g, k=70N/m, and b=50 g/s. What is the period of the motion?

Damped Oscillator

*Convert to SI units

(1kg/1000g)(300g) = .30kg

Because $b \ll (k/m)^{1/2} = 4.58$ kg/s, the period is approximately that of the undamped oscillator. Use the period equation

$T = 2\pi(m/k)^{1/2}$

$T = 2\pi(.30\text{kg}/70\text{ N/m})^{1/2} = .41\text{s}$

An unkown mass is oscillating on the pictured damped oscillator. The damping constant is 50 g/s. The spring constant is 60 N/m. The period is .51s. How long does it take for the amplitude of the damped oscillations to drop to half its initial value?

*First, solve for the unknown mass. Use the period formula.

$T = 2\pi(m/k)^{1/2}$

$.51\text{s} = 2\pi(m/60\text{N/m})^{1/2}$

Solve for m

m = .40kg

*Now, amplitude at time t is Xm^e bt/2m It has the value   Xm at t=0. Find the value of t for which
Xm^e bt/2m = Twenty-two points, plus triple-word-score, plus fifty points for using all my letters. Game's over. I'm outta here. 1/2Xm, divide by Xm and take ln of both sides.
Ln1/2 = bt/2m
Solve for t
(2mln1/2)/b = t = [2(.4kg)ln1/2]/.05kg/s
t = 11.1s

Amplitude is given by

Xm = Fm / [m^2(ùd^2-ù^2)^2 + b^{2}ùd^2]½

Where Fm is the constant amplitude of the external oscillating force exerted on the spring by the rigid support. At resonance, what is the amplitude of the oscillating object?

*Because during resonance ùd = ù, one can plug ù in for ùd.
Xm = Fm / [m^2(ù^2-ù^2)^2 + b^{2}ù^2]½
Xm = Fm / (b^{2}ù^2)½
Xm = Fm / bù