Chapter 3:

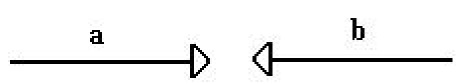
Vectors

This chapter provides the basis for the understanding and application of vectors that will be used in subsequent chapters throughout you text. Whether solving vectors using the graphical method or by combing their components through addition and multiplication, they serve as a valuable tool in unlocking the mysteries of the physical world. More importantly they shall be the key to passing your exams.

Vectors and Scalars

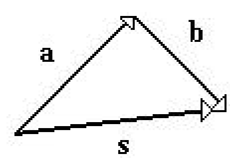
Synopsis

A particle's motion **vector** has magnitude as well as direction. Vector analysis moves the physics student out of the realm of “straight line” motion, and into the world of three-dimensional motion. Distinguishing between a vector quantity and a scalar quantity is an important skill for an observer to exhibit. Examples of scalar quantities are temperature, pressure, mass, and time. They do not point in a spatial sense. Force, velocity, and displacement are examples of vectors; they have a magnitude and direction. Below are illustrations of vectors **a** and **b**.



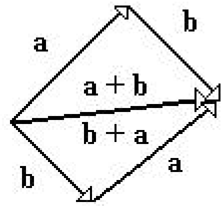
The Graphical Approach

Below are a few examples of how individual vectors can be combined to form various kinds of **vector sums**:



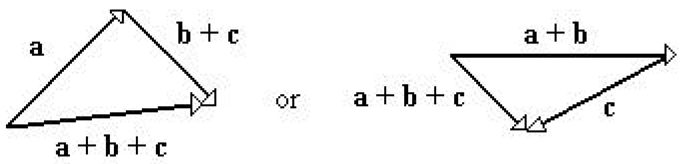
s = a + b
(**s** is referred to as the vector sum of **a** and **b**)

The commutative law, written **a** + **b** = **b** + **a**, explains that the order in which vectors are added to one another does not effect the resultant sum. For example:

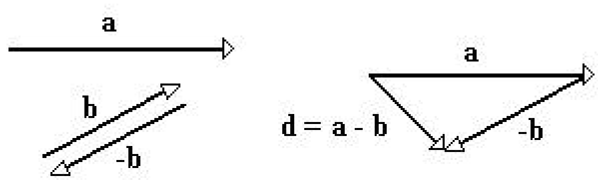


Also, vectors can be joined using the associative law which says that:

(**a + b) + c = a + (b + c)**, such that:



Subtracting a vector is the same as a adding a vector of equal magnitude to its original, but opposite in direction. Here is an illustration of how to subtract vectors from one another using the equation **d = a - b = a + (-b)**:



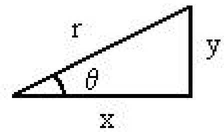
Tip: *When drawing two or more vectors, it is important to place the tail of the second vector at the head of the first and to draw every consecutive vector in this head-to-tail fashion. When constructing the* ***vector sum,*** *draw a vector from the tail of the first vector to the head of the last.*

The **major problem types** that you can expect from the above sections include vector addition and subtraction utilizing either the commutative or associative law or both.

Major concepts/equations

It is important to have a strong background in geometry. This will allow the student to add and subtract vectors graphically. Adding vectors graphically helps to simplify a concept, which could be quite confusing. The student will be able to construct drawings to assist in the solution of an otherwise impossible problem. Below are geometry concepts, which are important to master.

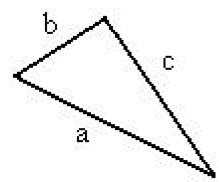
Trigonometric Functions Of Angle



sin= y/r   cos= x/r

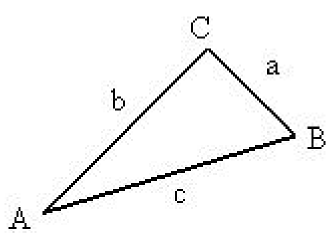
tan=y/x

Pythagorean Theorem



$a^2 + b^2 = c^2$

Triangles



Angles are A, B, C
Opposite sides are a, b, c
Angles A + B + C = 180 degrees

sinA/a = sinB/b = sinC/c

$c^2 = a^2 + b^2 - 2ab \cos C$

Vectors Components

When studying vectors it may be useful to break them up into their **component** parts. Take figure 1 for example.

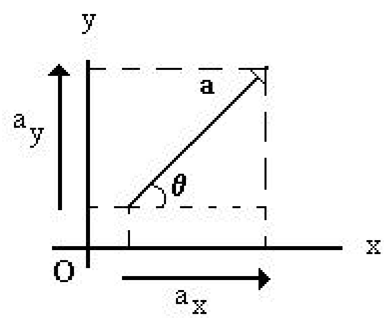


figure 1

Tip: *When drawing the components of a given vector place the tail end at the origin of a coordinate system to provide a more accurate representation.*

If we try to construct a vector from these parts, we refer to it as **resolving the vector**. To find the components of any vector you can use right triangle trigonometry with the following equations:

$$a_x = a \cos \theta \quad \text{and} \quad a_y = a \sin \theta$$

Understanding these components can help you in determining the magnitude and direction of a given vector by applying them to two simple equations:

$$a = \sqrt{a_x^2 + a_y^2} \qquad \text{and} \qquad \tan \theta = \frac{a_y}{a_x}$$

Tip: *When trying to find the angle of a vector from its components it use the inverse tangent* ($\tan^{-1}$).

Unit Vectors

A **unit vector** is a dimensionless quantity whose sole purpose is to specify a particular direction. Denoted as **i**, **j** and **k**, they are labeled below in Figure 2 along their corresponding axes.

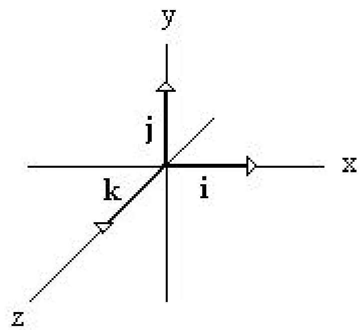


figure 2

Unit vectors can be assigned to **scalar components** in order to create **vector components**, such as in the following equations:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} \quad \text{and} \quad \mathbf{b} = b_x\mathbf{i} + b_y\mathbf{j}$$

Figure 3 shows their importance in vector diagrams:

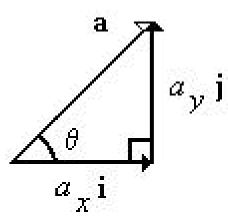


figure 3

Adding Vectors by Components

Major Concpets and Equations

To add two vectors **a** and **b**

$\mathbf{c} = \mathbf{a} + \mathbf{b}$

First break each vector (**a** and **b**) into components, then add the components of each vector to determine the compents of the resulting vector.

$\mathbf{c}_x = \mathbf{a}_x + \mathbf{b}_x$

$\mathbf{c}_y = \mathbf{a}_y + \mathbf{b}_y$

Then determine the magnitude of **c** by applying the pathagoren theorem to the copmonents.

$|\mathbf{c}|^2 = \mathbf{c}_x^{\,2} + \mathbf{c}_y^{\,2}$

Next the direction of **c** must be determined. To find the angle O of either the x or y axis take the iverse tangent of the opposite component divided by the adjacent component. So the angle from the x axis would be

$\theta = \tan^{-1}(\mathbf{c}_y / \mathbf{c}_x)$

And the angle from the y axis would be

$\theta = \tan^{-1}(\mathbf{c}_x / \mathbf{c}_y)$

If the vectors **a** and **b** are given in unit vector notation adding the vectors is very simple. If the vectors are given in the form

$\mathbf{a} = n\boldsymbol{i} + m\boldsymbol{j}$

$\mathbf{b} = p\boldsymbol{i} + q\boldsymbol{j}$

Then simply add the components in the ***i*** direction of each vector and add the components in the ***j*** direction of each vector.

$\mathbf{c} = (n\boldsymbol{i} + p\boldsymbol{i}) + (m\boldsymbol{j} + q\boldsymbol{j})$

Major Porblem Types

Net Displacement
Net Velocity
Net Force
etc.

Tips and Advice

Adding vectors by components if very simple. Always remember to put the vectors on an x,y axis and find the components of each vector on that axis. Also don’t forget to find both magnitude and the direction of the resulting vector.

Vectors and the Laws of Physics

Synopsis: Coordinate systems are in multitude when it comes to vectors. There is an infinite number of coordinate systems for each and every situation in physics. Relations among vectors do not depend upon location of the origin or orientation of the axes.

Multiplying Vectors

3 ways in which vectors can be multiplied: (1) Vector by scalar
(2) Vector by vector (scalar/dot product)
(3) Vector by vector (vector/cross product)

VECTOR BY SCALAR:

When you multiply a vector by a scalar you get a vector with a magnitude of the product of the original vector and the absolute value of the scalar. The direction of the resultant vector-scalar product is the direction of the original vector if the scalar is positive, and opposite the direction of the original vector if the scalar is negative. If the scalar is a physical quantity the physical nature of the product of the vector and the scalar differs from that of the original vector.

VECTOR BY VECTOR (scalar/dot product):

- **a b =** ab cos θ; where a is the magnitude of vector **a**, and b is the magnitude of vector **b**, and is the angle between **a** and **b**.
- The dot product is the product of the magnitude of one vector and the scalar component of the second along with the direction of the first vector.
- If is 0, then the dot product is a maximum
- If is 90, then the dot product is zero
- If is 180, then the dot product is a minimum

- **a b=** (**a** cos θ) **b= a** (**b** cos θ)
- **a b= b a (commutative law)**

The Vector Product (Cross Product)

- **c=** ab sin θ; where is the smaller of the two angles between **a** and **b**
- If **a** and **b** are parallel to each other, **a b= 0;** If **a** and **b** are perpendicular

to each other, **a b = 1**

- • Direction of **c** is established by the “right-hand rule”
- • **Right-Hand Rule:**
  1. place vector **a** and **b** tail to tail
  2. grasp the line that is perpendicular to **a** and **b** with your right hand
  3. sweep **a** into **b** through
  4. your thumb will point to **c**

  REMEMBER: **b a = -(a b) (commutative law does not apply)**

Tips for Multiplication of Vectors:

- • The key to vector multiplication is, knowing the rules associated with dot products and cross products. These important relations will allow you to fully analyze the products.
- • When multiplying vectors, always be sure to pair all components, and then analyze the vector resultant component pairs at the end of the multiplication, based on whether a dot or a cross product is being found.
- • **Relations to Know: (when i, j, and k are unit vectors in the** x, y, and z **directions)**

  When doing dot products, remember: **i j=0; j k=0; and k i=0**
  ii= jj= kk=1
  (Also, remember that the dot product obeys the commutative law)

  When doing cross products remember: **ii= jj= kk= 0**
  ij=k; jk=i; ki=j

  (Remember that cross products do not obey the commutative law)
  so: ji= -k; kj= -i; ik= -j

- **Tip:** *When slaving over vector multiplication problems, just remind yourself of one thing: “vectors are my friend.”*

Links

Here are a few links that may help you out if you run into a few problems:

www.ping.be/math/vectors.htm

www.cut-the-knot.com/do_you_know/mul_scal1.html

Web Page Authors: Joe Bledowski, Sam Scott, Terence Lantham, and Charles Neering

Bibliography

Halliday, David, Robert Resnick, and Jearl Walker. Fundamentals of Physics: 5th Edition. New York: John Wiley & Sons, Inc., 1997.