Chapter 4 Motion in Two and Three Dimensions

Daniel Bartels, Stacie Boston, Tara George, Waverly Jones, Charles Sims

Synopsis

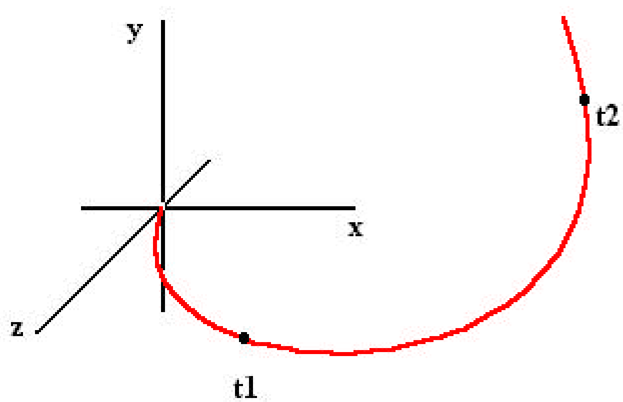
We combine and extend the concepts of the previous two chapters (vectors and 1d motion) to two and three dimensions, then apply them to problems in two dimensions namely projectile and uniform circular motion. The concept of relative motion is also introduced.

Major concepts and equations

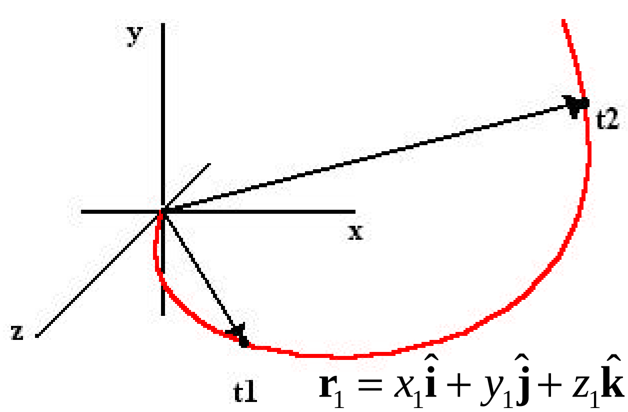
Position and Displacement

A particle's path and its position at two points in time

t1(x1,y1,z1), t2(x2,y2,z2)



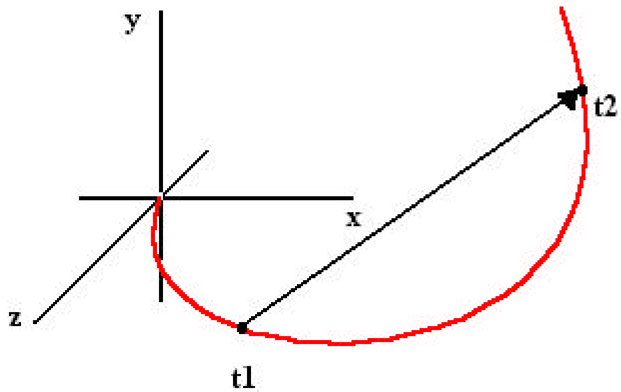
Position can also be represented by a vector from an arbitrary origin to the point



$$\mathbf{r}_2 = x_2\hat{\mathbf{i}} + y_2\hat{\mathbf{j}} + z_2\hat{\mathbf{k}}$$

$$\mathbf{r}_1 = x_1\hat{\mathbf{i}} + y_1\hat{\mathbf{j}} + z_1\hat{\mathbf{k}}$$

The displacement vector is the difference between the position vectors
Directed from the first point in time to the second



$$\Delta \mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$$

$$\Delta \mathbf{r} = (x_2 - x_1)\hat{\mathbf{i}} + (y_2 - y_1)\hat{\mathbf{j}} + (z_2 - z_1)\hat{\mathbf{k}}$$

<u>Velocity and Average Velocity</u>

Average velocity is the change in position over the change in time

$$\bar{\mathbf{v}} = \frac{\Delta \mathbf{r}}{\Delta t}$$

Instantaneous velocity, or simply velocity, is the time derivative of position and is always tangent to the path of the particle

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

Acceleration and Average Acceleration

Average velocity is the change in velocity over the change in time

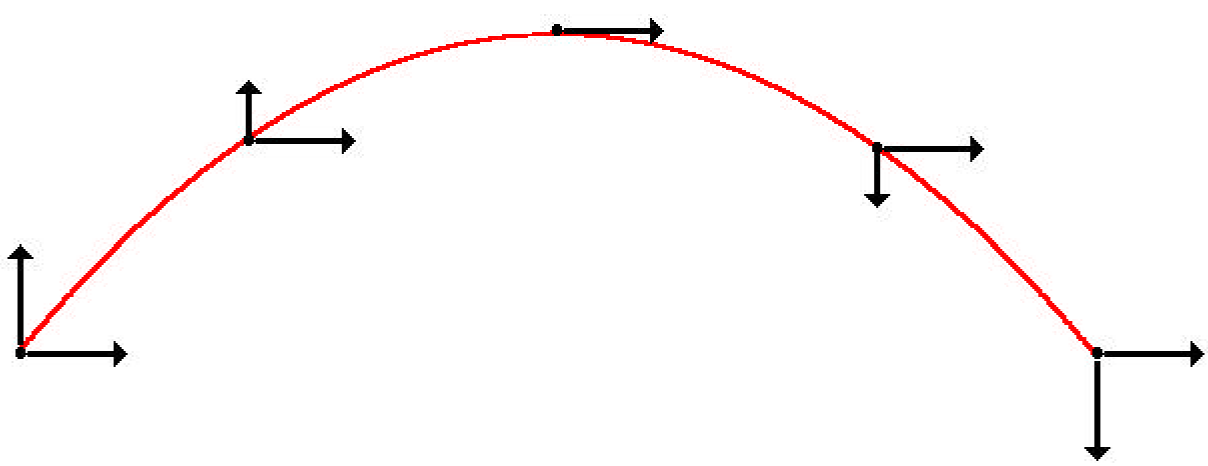
$$\bar{\mathbf{a}} = \frac{v_2 + v_1}{\Delta t} = \frac{\Delta \mathbf{v}}{\Delta t}$$

Instantaneous acceleration, or simply acceleration, is the time derivative of velocity

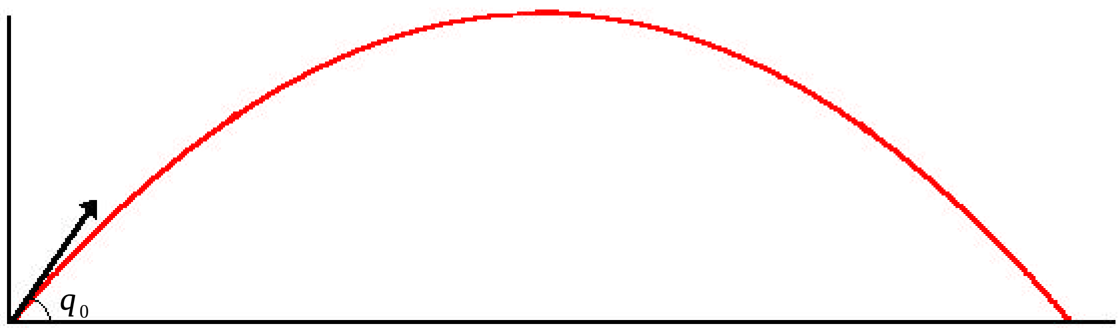
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\hat{\mathbf{i}} + a_y\hat{\mathbf{j}} + a_z\hat{\mathbf{k}}$$

Projectile Motion

Horizontal and vertical components of motion are not dependent on one another
The horizontal component of velocity is constant – zero acceleration



Assume the vector is in the first quadrant and ***q*** is the angle between the vector and the x- axis.



There is <u>no</u> acceleration affecting the x-component:

$$\Delta x = (v_0 \cos q_0)t$$

Gravity affects only the y-component:

$$\Delta y = (v_0 \sin q_0)t - \tfrac{1}{2} g t^2$$

$$v_y = v_0 \sin q_0 - gt$$

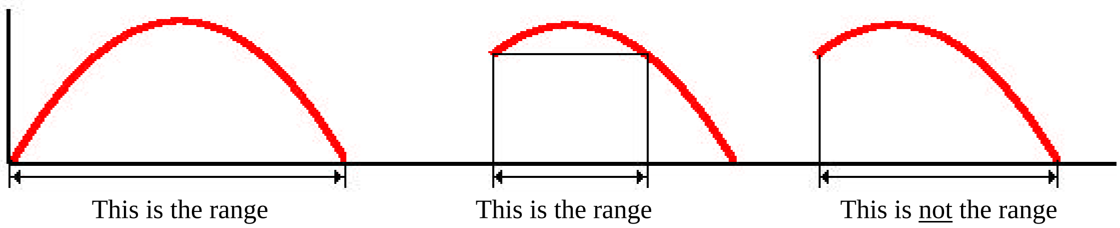
$$v_y^2 = (v_0 \sin q_0)^2 - 2gt\Delta y$$

The equation of path:

$$y = (\tan q_0)x \frac{gx^2}{2(v_0 \cos q_0)^2}$$

The range ***R*** of a projectile is the horizontal distance between the projectile's launching point and the point at which it returns to the launch height.

$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

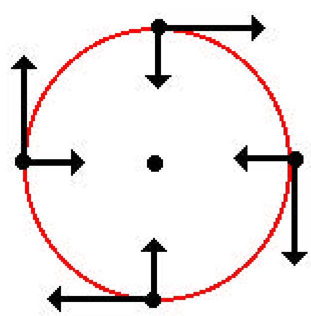


The range is maximized when the launch angle is 45° (sin 2(45°) = sin 90° = 1).

Uniform Circular Motion

The *magnitude* of velocity (speed) is constant but its direction constantly changes ∴

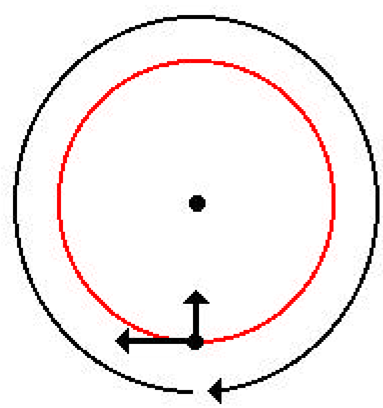
The particle is accelerating.



The acceleration is always towards the center (centripetal) and has magnitude:

$$|\mathbf{a}| = a = \frac{v_0^2}{r}$$

The period of revolution T – the time to travel the circumference.



$$T = \frac{2pr}{v}$$

<u>Relative Motion in One Dimension</u>

There is no fundamental or underlying coordinate system for the universe –
Any observer is justified in choosing any arbitrary origin for their own coordinate system or **reference frame**.

The velocity of **P** with respect to **A** is equal to the velocity of **P** with respect to **B** plus the velocity of **B** with respect to **A.**

$$\mathbf{v}_{PA} = \mathbf{v}_{PB} + \mathbf{v}_{BA}$$

Inertial frames of reference have constant velocity.

The acceleration of a particle is constant relative to observers on different inertial frames of reference.

$$\mathbf{a}_{PA} = \mathbf{a}_{PB}$$

Problem Types

- **Projectile motion**

The path of the projectile is parabolic and generally falls into one of the three classes:

$\Delta y = 0$ $\Delta y > 0$ $\Delta y < 0$

We are commonly asked to find the following things:

The range of the projectile.

How long it was in the air.

Its maximum height.

Δy

Its velocity at some point.

The line of sight required to strike a target when dropped (launched) from some airborne object.

- **Uniform circular motion**

We are given some (usually two) of the following quantities and asked to find another:

the period of revolution

the magnitude of acceleration

the radius of the circle or arc

the magnitude of velocity

- **Relative Motion in One Dimension**

We are generally given two observers on different inertial frames of reference and some pointlike object. Given data about two of the three we determine the velocity and/or acceleration of the third. Using that we may be asked to find another quantity using ‘standard’ motion formulas.

Tips and Advice

- **General**
  - Restate the problem
    - Eliminate unnecessary verbiage
    - Look for key concepts
    - Reduce it to a more concise statement with just the facts
  - Make a rough sketch
  - List what’s given and/or implied
    - Eliminate unnecessary data
    - Convert units to a common system
    - Transform data into useful form **-** resolve vectors
  - Refine the sketch
    - Label the sketch with what you know
  - Determine what’s being asked for
    - Is it just one problem or a series of problems?
    - If it’s a series is it explicit or implicit?
      - If it’s explicit (it has parts a), b), c), etc.) realize that the order they’re in isn’t necessarily the order in which to tackle them. It might be b) then c) then a).

        - If it's implicit divide and conquer. Break it into smaller problems, solve them then put it all together
    - List the quantities that you're looking for

- Refine the sketch
- Determine the appropriate formula(s)

    - Keep and regularly update a list of formulas

- Solve symbolically <u>then</u> numerically

    - It's easier to manipulate symbols than numbers with units
    - Less error due to rounding
    - How the quantities relate is more apparent

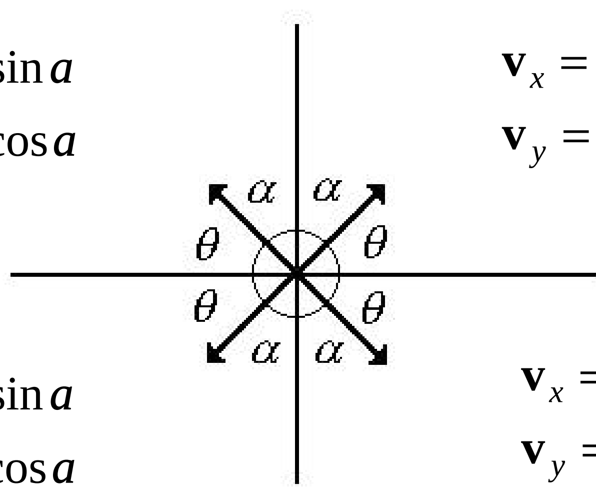
- Does the answer make sense?

    - If a mistake has been made it <u>will</u> show up in the answer
    - If the problem is set up wrong but the units have been correctly cancelled, the units of the resulting answer probably won't be the units of the quantity you're looking for.

Resolving vectors is essential to the solution of many problems including those of projectile motion. Once the vector is resolved into components treat the problem as two separate problems – one for each component.

$$\mathbf{v}_x = -v\cos q = -v\sin a$$
$$\mathbf{v}_y = +v\sin q = +v\cos a$$

$$\mathbf{v}_x = +v\cos q = +v\sin a$$
$$\mathbf{v}_y = +v\sin q = +v\cos a$$



$$\mathbf{v}_x = -v\cos q = -v\sin a$$
$$\mathbf{v}_y = -v\sin q = -v\cos a$$

$$\mathbf{v}_x = +v\cos q = +v\sin a$$
$$\mathbf{v}_y = -v\sin q = -v\cos a$$

LINKS

Instead of providing many links one is listed that contains numerous links to many useful sites

Eric's Treasure Trove: http://www.astro.virginia.edu/~eww6n/physics/physics.html