Chapter 7 deals with **kinetic energy** and **work**, with the major concept being

$$\Delta K = W$$

Kinetic energy (*the energy of motion*) is expressed in terms of joules, where 1 joule is equivalent to 1 kg-m^2/s^2. It is defined by the formula

$$K = \frac{1}{2} mv^2$$

The only component of K that may change is velocity. K is directly related to velocity. This is why kinetic energy is the energy of motion.

$$\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - \frac{1}{2} mv_i^2 = W$$

There are alternative ways to express work and kinetic energy, such as the Newton-meter (Nm) and the foot-pound (ft-lb). When dealing with atomic particles, the common unit of energy is the electron-volt (eV). 1 eV = 1.6×10^{-19} J.

Work Done by Constant Forces

Work, which is a transfer of energy by a force(s), is also expressed in joules.

$$W = Fd \cos \phi$$

where ϕ represents the angle between the force and the direction of the object's motion, and d represents the distance over which the force is applied. Hence, when a force is applied to an object at an angle of 90°, it does no work on that object (cos 90° = 0). Similarly, forces applied at angles of 0° and 180° do +W and –W on an object, respectively. *Friction* is an example of a force that does –W on an object, since it always opposes the motion of an object. When $\Delta K > 0$, W is positive, and it caused the object's velocity to increase. When $\Delta K < 0$, W is negative. Negative work done on an object will cause the object's velocity to decrease.

If several forces are applied to an object, then the total work done on the object is

$$W_{TOT} = F_1 \bullet d + F_2 \bullet d + F_3 \bullet d + \ldots = \sum F \bullet d$$

Note that each $F_n \bullet d$ equals a W, so the total work done on an object is the sum of all of the works produced by each force.

Work Done by Weight

The *weight* of an object will also do work on that object. Instead of using $W = Fd \cos \phi$, we replace F with mg and obtain

$$W_{mg} = mgd \cos \phi$$

This makes sense since the units for F are kg-m/s^2, and mg = kg-m/s^2. When an object is rising, its weight is doing negative work on it because the motion of the object is up, while its weight is directed down. This 180° difference accounts for the negative sign of the work (cos 180° = -1). The exact opposite is true when an object is falling. Here, its weight is pulling the object down. It is acting in the same direction as the object's motion (0° difference) so it is doing positive work.

We will often encounter problems where we either lift or lower an object. When lifting the object, the applied force transfers energy to the object. Lowering it is just the opposite, and the applied force transfers energy from the object. The general equation for these scenarios is

$$\Delta K = W_{APP} + W_{mg}$$

When an object is lifted from a *resting position* and ends up in a *resting position*, its $\Delta K = 0$. This means that the applied force did work that was opposite in direction but the same magnitude as the work done by the object's weight. Hence,

$$W_{APP} - W_{mg} = 0$$

Work Done by Variable Forces

Since forces are not always constant, we must also address the work done by variable forces on an object. We consider *only forces which change in magnitude, not direction*. The magnitude changes with the object's position. Our equation for work then becomes

$$W = \int_{x_i}^{x_f} F(x)dx$$

To find the total work done in three dimensions, we simply apply the same equation to the x, y and z dimensions and add the results of each integral.

Spring Force

A third type of force that can do work on an object is that of a spring force. We measure this force by the formula

$$F = -kd$$

where k is the spring constant (the stiffness of the spring measured in N/m) and d is the distance that the spring's free end is displaced from its relaxed state. *The spring force is a variable force*, so we cannot use the W = F•d equation. The work done by a spring is expressed by:

$$W_{SPRING} = \tfrac{1}{2} kd_i^2 - \tfrac{1}{2} kd_f^2$$

Graphing

When dealing with a force doing work on an object, if we graph F(x) versus x (where x = the object's displacement), and then choose two points on the displacement axis, the area bounded by the F(x) curve and the displacement axis constitutes the amount of work performed during the displacement interval. Variable forces will require one to solve an integral in order to find this area, since F(x) will not be the simple horizontal line which is the trademark of a constant force.

Power

Quite often, we wish to express the rate at which work is done by a force(s), or the rate at which an applied force(s) transfers energy to an object. We want to know how many *joules per second* of work is being done. Rather than stating it this way, we normally use units such as watts or horsepower to convey this rate. Whether it's watts, horsepower, or ft-lbs/s, we are speaking of *power* in general. The equations which obtain the average power and instantaneous power due to a force(s) are

$$P_{AVG} = \frac{W}{\Delta t} \qquad \text{AND} \qquad P = \frac{dW}{dt}$$

Instantaneous power may also be expressed as

$$P = F \bullet v$$

Common Conversions

1 eV = 1.6×10^{-19} J
1 J = 0.738 ft-lb
1 horsepower = 550 ft-lb/s
1 horsepower = 746 W
1 W = 1 J/s
1 calorie = 4.19 J

Tips / Advice

K can never be negative!
Note your signs!
Note the direction of all forces so that you may apply the correct sign when determining the work being done by each of them.
Note the direction that the object is moving.
Determine whether or not the forces are constant or variable.
Note the initial and final velocities of the object.
Be sure to account for all forces acting on the object, including gravity (mg).