Chapter 9 – Systems of Particles

Main topics:

- Center of Mass
- Newton's 2nd Law For a System of Particles
- Linear Momentum/Conservation of
- Systems of Varying Mass

Center of Mass

The Center of mass a point at which you can treat any size object as a point mass when applying external forces.

In a Boomerang, the center of mass is located in it's internal concavity.

The formula for the center of mass is given by

$$\mathbf{r}_{cm} = \frac{1}{M}\sum_{i=1}^{n} m_i\mathbf{r}$$

- Tips: When working with center of mass problems, break **r** down into it's x, y, and z components and remember that the center of mass is not always in the object.

Newton's 2nd Law for a System of Particles

$$\sum \mathbf{F}_{ext} = M\mathbf{a}_{cm}$$

Where

1. $\sum \mathbf{F}_{ext}$ is the vector sum of all the external forces acting on the system. Always be sure not to include internal forces, or forces that one part of a system exerts on another part.

2. M is the total mass of the system and is constant.

3. $\mathbf{a}_{cm}$ is the acceleration of the center of mass of the system and not any other point in the system.

Linear Momentum/Conservation of

Linear momentum (a vector quantity) of a particle is defined as

$$\mathbf{p} = m\mathbf{v}$$

Or momentum is equal to the mass times the velocity. Its SI units are kilogram-meter per second.

For a system of particles, it simply changes to

$$\mathbf{P} = M\mathbf{v}_{cm}$$

Using momentum, Newton's second law can be written as

$$\sum \mathbf{F}_{ext} = \frac{d\mathbf{P}}{dt}$$

In any closed, isolated system, the law of conservation of linear momentum states that

$$\mathbf{P}_i = \mathbf{P}_f$$

Question: If a system of particles has zero momentum does the system necessarily have zero kinetic energy?

If $\mathbf{P} = M\mathbf{v}_{cm} = 0$ does $KE = \frac{1}{2}mv^2 = 0$?

NO! Momentum is a vector quantity, while kinetic energy is a scalar.

- Tips: Make sure the system is closed and isolated. Be careful that boundaries of the system exclude objects that exert non-conservative forces. In other words, in that case Pi would not equal Pf.

Systems with Varying Mass

Things get a little more complicated when the mass begins to vary, like in a rocket for example. The book gives two equations for working with rockets.

First Rocket Equation says

$$Ru = Ma$$

Where R is the rate at which the fuel is consumed and u is the speed at which it is being ejected. Thrust, T, is defined as Ru, giving you

T = Ma

Where a is the acceleration at whatever time it's mass is M.

Second Rocket Equation says

$$v_f - v_i = u \ln \frac{M_i}{M_f}$$

With Mi as the initial mass and Mf as the final mass

- Tips: Systems with varying mass are characteristic of rocket problems.

The shuttle uses thousands of tons of fuel
to place only a few tons of material into orbit.

Links:

Test your skills with momentum conservation with a game of pool:
http://members.aol.com/jiping/pool.html
University of Wisconsin discussion of Linear Momentum/C.M.
http://lupine.physics.wisc.edu/207/lecture10/image-index.html
University of Illinois at Urbana-Champaign - Physics notes
http://courses.physics.uiuc.edu/cyberprof-docs/physics/phys101/lect/