Problem 1	Problem 2	Problem 3	Problem 4	Bonus	% Grade

Name: Key **SS#**

PHYSICS 207E MIDTERM EXAM II

November 18, 1999

Please Circle or **underline** your answers. SHOW ALL WORK!

"On my honor, I have neither given nor received aid on this assignment."

Student's Signature: ____________________

Bonus Question (5 points)

Consider the potential energy function

$$U(x) = 2x^3 - 3x^2 \text{ (J)}$$

(a) Plot U(x) versus x for x>0
(b) Find the Force F(x) and plot it on the same graph
(c) Are there any points of stable equilibrium?

(a) $U(x) = 2x^3 - 3x^2 = x^2[2x-3]$

$\Rightarrow U(x) = 0$ at $x = 0$, $x = 3/2$ m

(b) $F(x) = -\dfrac{dU(x)}{dx} = -(6x^2 - 6x)$

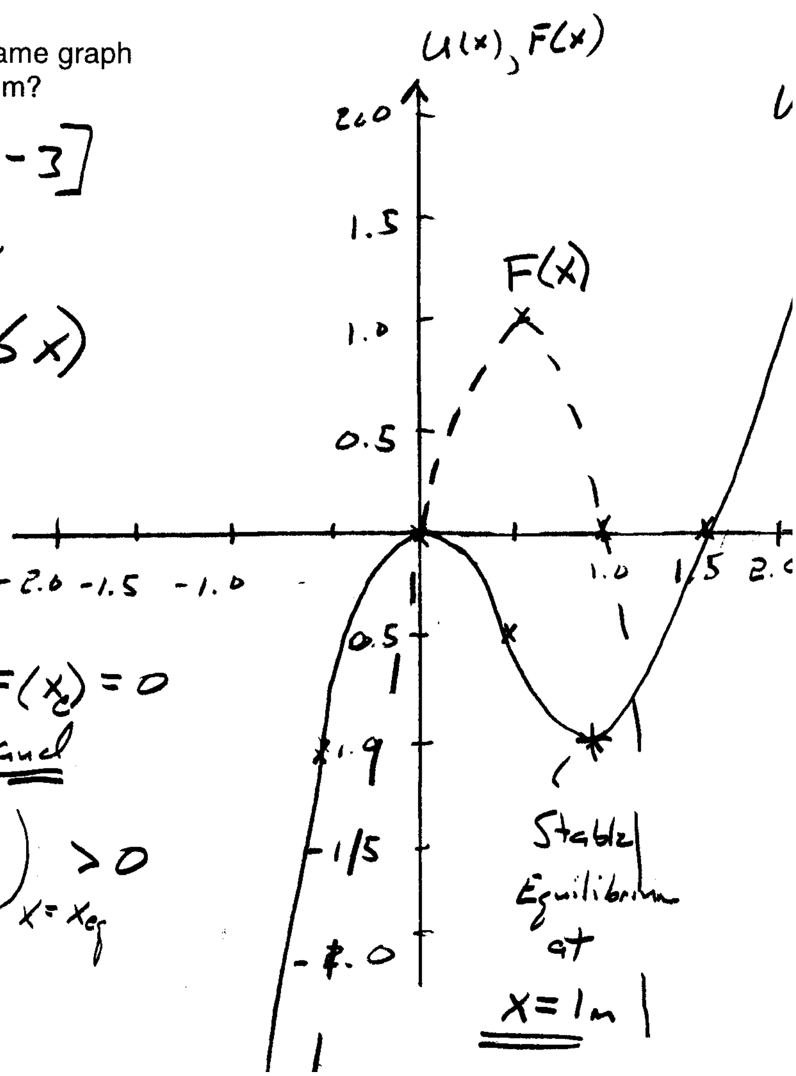
$= -6x[x-1]$

$F(x) = 0$ at $x = 0$, $x = 1$ m

(c) Stable Equilibrium $\Rightarrow$ $F(x_{eq}) = 0$ and $\left.\dfrac{d^2U}{dx^2}\right)_{x=x_{eq}} > 0$

$\dfrac{d^2U}{dx^2} = 12x - 6$

$\left.\dfrac{d^2U}{dx^2}\right)_{x=1} = 6 > 0$



1. A 4.0-kg block hangs by a light string that passes over a massless, frictionless pulley and is connected to a 6-kg block that rests on a rough shelf. The coefficient of kinetic friction is 0.2. The 6.0-kg block is pushed against a spring, to which it is NOT attached. The spring constant is 180 N/m, and it is compressed 30 cm. Your task is to find the speed of the blocks after the spring is released and the 4.0-kg block has fallen a distance of 40 cm.

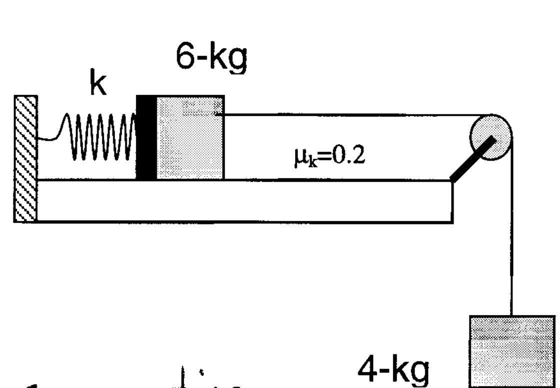
(a) Discuss using words and equations how you will approach solving this problem.

(b) Find the initial potential energy of the system.

(c) Find the energy dissipated by friction.

(d) Find the speed of the blocks after the 4-kg block has dropped a distance of 40 cm.

(e) Find the tension in the string.



(a) Use $\Delta E = W_{nc}$ Since friction is non-conservative.

(b) Set $U = 0$ at point where 4-kg mass has fallen 40 cm.

$$\Rightarrow U_1 = U_{spring} + U_{gravity} = \frac{1}{2}kx^2 + mgh$$

$$= \frac{1}{2}(180\ \text{N/m})(0.30\ \text{m})^2 + (4.0\ \text{kg})(9.8\ \text{m/s}^2)(0.40\ \text{m})$$

$$= 8.1\ \text{J} + 15.7\ \text{J} = \underline{\underline{23.8\ \text{J}}}$$

(c) $W_{nc} = -\mu_k mgd = (0.2)(6.0\ \text{kg})(9.8\ \text{m/s}^2)(0.40\ \text{m}) = \underline{\underline{-4.70\ \text{J}}}$

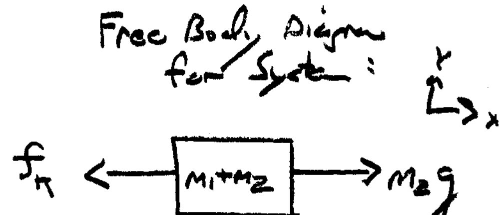
(d) $\Delta K + \Delta U = W_{nc}$ $U_1 = 23.8\ \text{J}$ $U_2 = 0$

$K_1 = 0$ $K_2 = \frac{1}{2}(m_1+m_2)V^2$

$$\Rightarrow \frac{1}{2}(m_1+m_2)V^2 + (-23.8\ \text{J}) = -4.70\ \text{J} \quad \therefore \underline{\underline{V = 2.0\ \text{m/s}}}$$

(e) The Tension T after the block moves away from the spring is

Free Body Diagram for System: F.B.D. for m



$$T = m_2 g - m_2\left[\frac{m_2 g - \mu_k m_1 g}{m_1+m_2}\right]$$

$$m_2 g - \mu_k m_1 g = (m_1+m_2)a$$

$$= m_2 g\left[1 - \frac{m_2 - \mu_k m_1}{m_1+m_2}\right]$$

$$\therefore a = \frac{m_2 g - \mu_k m_1 g}{m_1+m_2}$$

$m_2 g - T = m$

$T = m_2 g -$

$$= (4\ \text{kg})(9.8\ \text{m/s}^2)\left[1 - \frac{(4\ \text{kg}) - (0.2)(6\ \text{kg})}{(4\ \text{kg}) + (6\ \text{kg})}\right] = \underline{\underline{28.2\ \text{N}}}$$

Name: Key

2. A Styrofoam ball of mass M hangs at rest from a peg on a massless string of length R. A BB pellet of mass m strikes and sticks inside the ball. Your task is to find the minimum velocity $\mathbf{v}$ that the pellet must have, so that the pellet+ball will swing counterclockwise and complete a vertical circle. [Hint: The speed of the pellet+ball at the top of the circle will not be zero, but such that the tension in the string at that time is zero.]

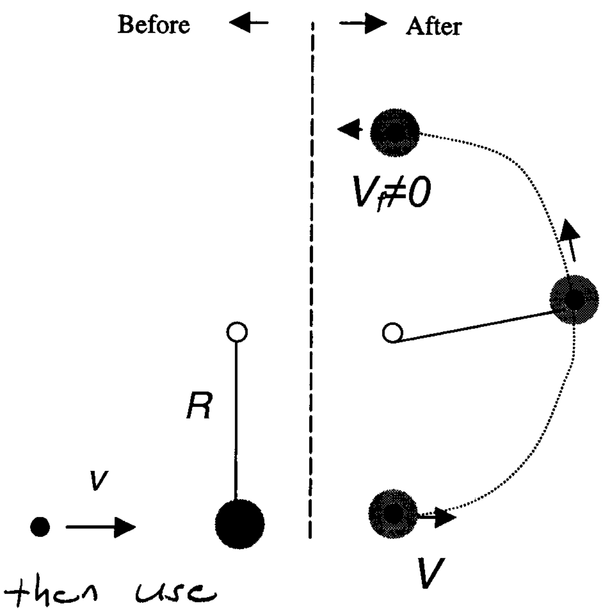
(a) Briefly describe using words and equations how you are going to approach solving this problem.

(b) Find an expression for the speed ($\mathbf{V}$) of the pellet+ball after the collision.

(c) Draw a free-body diagram showing the forces on the pellet+ball at the top of the circle.

(d) Find an expression for the speed $\mathbf{V_f}$ of the pellet+ball at the top of the circle.

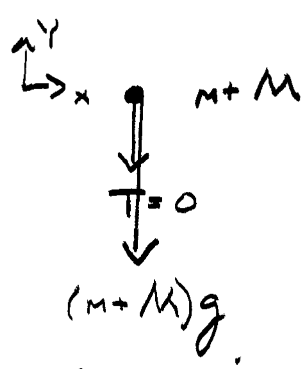
(e) Find an expression for the minimun velocity of the pellet ($\mathbf{v}$).



(a) Use $P_i = P_f$ for the collision, then use Conservation of Energy $\Delta E = 0$

(b) $P_1 = mv \quad P_2 = (m+M)V \implies \underline{\underline{V = \frac{mv}{m+M}}}$

(c)



(d) Uniform circular motion:

$$\sum F_y = T + (m+M)g = \frac{(m+M)V_f^2}{R}$$

Since $T = 0 \implies \underline{\underline{V_f = \sqrt{gR}}}$

(e) Now use $E_1 = E_2$

$$K_1 = \frac{1}{2}(m+M)V^2 = \frac{m^2 v^2}{2(m+M)}$$

$$K_2 = \frac{1}{2}(m+M)V_f^2 = \frac{1}{2}(m+M) \cdot g$$

$$U_1 = 0 \qquad U_2 = (m+M) \cdot g \cdot (2R)$$

$$\frac{m^2 v^2}{2(m+M)} = \frac{1}{2}(m+M)gR + (m+M$$

$$m^2 v^2 = 5(m+M)^2 gR$$

$$\implies \underline{\underline{v = \sqrt{5gR} \cdot \left(\frac{m+M}{m}\right)}}$$

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3. A 3.0-kg bomb slides along a frictionless horizontal plane in the x direction at 6 m/s. It explodes into two pieces, one of mass 2.0-kg and the other of mass 1.0-kg. After the explosion, the 1.0 kg piece moves in the horizontal plane in the y direction at 4.0 m/s. Your task is to find the velocity of the 2.0-kg piece after the explosion, the velocity of the center of mass, and the energy released.

(a) Draw a nice before/after picture of this problem.

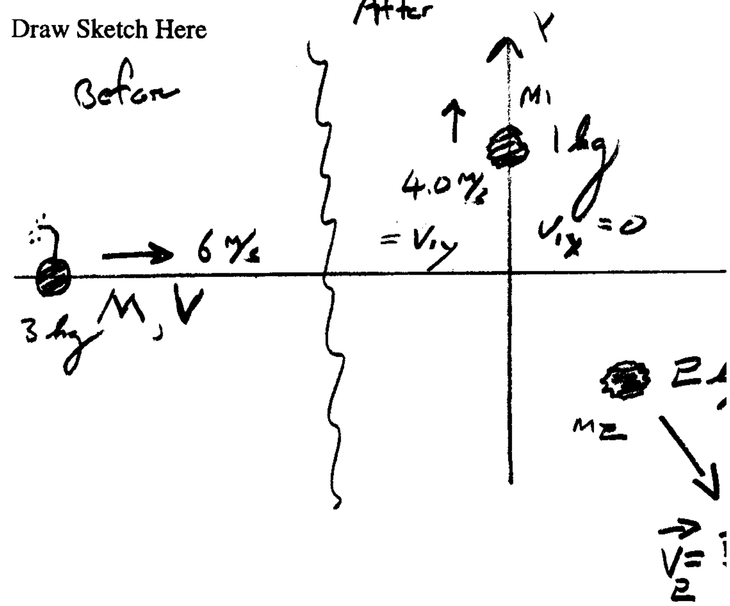
(b) Describe using words and equations how you are going to approach solving this problem

(c) Find the velocity of the 2-kg piece after the explosion.

(d) Find the velocity of the center-of-mass after the explosion.

(e) Find the energy released in the explosion.

Draw Sketch Here



(b) Use Conservation of momentum

$$\vec{P}_1 = \vec{P}_f \quad (\text{vector form})$$

(c) $P_{1x} = MV \qquad P_{1y} = 0 \qquad \vec{P}_1 = MV\hat{i}$

$P_{2x} = m_2 v_{2x} \qquad P_{2y} = m_1 V_{1y} + m_2 v_{2y} \qquad \vec{P}_2 = m_1 V_{1x}\hat{i} + m_2 V_{2y}\hat{j} + m_2$

$V_{2x} = \frac{M}{m_2} V \qquad V_{2y} = -\frac{m_1}{m_2} V_{1y} \implies \underline{\underline{\vec{V}_2 = 9\,\text{m/s}\,\hat{i} - 2\,\text{m/s}\,\hat{j}}}$

or

$V_{2x} = +9.0\,\text{m/s} \qquad V_{2y} = -2\,\text{m/s} \qquad V_2 = \underline{\underline{9.2\,\text{m/s}}},\ \underline{\underline{-12.5^\circ}}$ from $+$

(d) Since $\sum F_{ext} = 0 = m\vec{a}_{cm} \implies V_{cm}^{i} = V_{cm}^{f} = \underline{\underline{6\,\text{m/s}}}$

(e) $K_I = \frac{1}{2}MV^2 = \underline{\underline{54\text{ J}}} \qquad K_f = K_{1f} + K_{2f} = \frac{1}{2}m_1 V_1^2 + \frac{1}{2}m_2 V_2^2$

$= 92.6\text{ J}$

$\Delta K = K_f - K_I = \underline{\underline{38.6\text{ J}}}$

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4. A uniform sphere of mass *M* and radius *R* is free to rotate about a horizontal axis through its center. A string is wrapped around the sphere and is attached to a block of mass *m* as shown. Your task is to find the acceleration of the object, the tension in the string, and the angular velocity of the sphere after the block has dropped a distance *h*. [$I=(2/5)MR^2$, and remember that a and α are related]

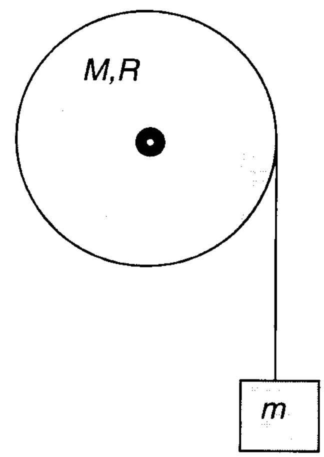
(a) Draw free-body diagrams showing the Forces on the sphere and the block.

(b) Describe using words and equations how you are going to approach solving this problem.

(c) Find the acceleration of the block.

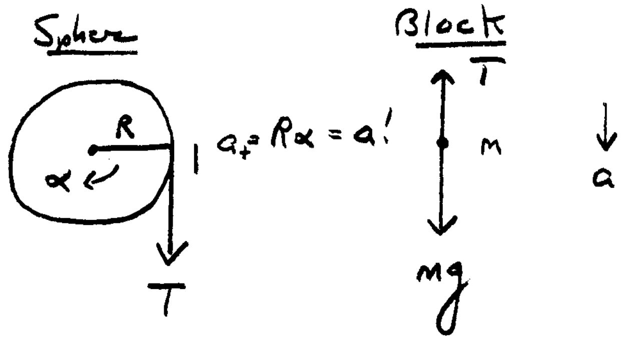
(d) Find the tension in the string.

(e) Find, **using energy methods**, the angular velocity of the sphere after the block has dropped a distance *h* from its original height.



Draw force diagrams here

(a)



(b) Use Newton's 2nd Law in Rotational form: $\sum \tau = I\alpha$ then $\Delta E = 0$
(Also need $\sum F = ma$ for the Block)

(c) Sphere

$\sum \tau = I\alpha$

$\Rightarrow TR = \frac{2}{5}MR^2\alpha$

Block

$\sum F = ma = mg - T$

$\Rightarrow T = m(g-a)$

Since $\alpha = a/R$, we have, Subin's for T:

$m(g-a) = \frac{2}{5}Ma$

$mg = \left(\frac{2}{5}M + m\right)a$

$$a = \frac{mg}{m + \frac{2}{5}M}$$

(d) $T = mg - \dfrac{m^2 g}{m + \frac{2}{5}M}$

$T = mg\left[1 - \dfrac{m}{m + \frac{2}{5}M}\right]$

$$T = mg\left[\frac{2M}{5m + 2M}\right]$$

(e) $\cancel{K_1}^{0} + U_1 = K_2 + \cancel{U_2}^{0}$

$U_1 = mgh$

$K_2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

use $v = R\omega$, $I = \frac{2}{5}MR^2$

$mgh = \frac{1}{2}mR^2\omega^2 + \frac{1}{5}MR^2\omega^2 = \left(\frac{1}{2}m + \frac{1}{5}M\right)R^2\omega^2$

$$\therefore \quad \omega = \sqrt{\frac{10mgh/R^2}{5m + 2M}}$$

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